

EXAM INTRO TO MATH ('MODULAR ARITHMETIC')  
07-11-2017, 2:00PM-3:00PM (ALETTA JACOBS HAL 01)

THE EXAM CONSISTS OF TWO PROBLEMS,  
YOU CAN SCORE IN TOTAL 10 POINTS.

USING A SIMPLE CALCULATOR DURING THE EXAM IS ALLOWED.

[1 point.]  $\left\{ \begin{array}{l} \text{Name:} \\ \text{Student Number:} \\ \text{Number of sheets handed in:} \end{array} \right.$

- (1) Suppose  $a$  and  $b$  are integers not both zero, and  $d := \gcd(a, b)$ . Let  $D(a, b)$  be the set of all divisors of  $d$ , and let  $C(a, b)$  be the set of all common divisors of  $a$  and  $b$ .
- (a) [1 point.] Show that  $d$  is in the set  $C(a, b)$ .
  - (b) [1 point.] Show that all elements of  $D(a, b)$  are also contained in  $C(a, b)$ .
  - (c) [1 point.] Show that all elements of  $C(a, b)$  are also contained in  $D(a, b)$ .
- (2) For  $n \geq 0$  we introduce the number  $g_n = (19 \cdot 10^n - 1)/9$ .
- (a) [1 point.] Show that  $g_n$  is an integer, for any  $n \geq 0$ .
  - (b) [1 point.] Show that  $g_{n+1} = 10g_n + 1$ , for any  $n \geq 0$ .
  - (c) [1 point.] Show that  $g_n \bmod 100 = 11 \bmod 100$ , for every  $n \geq 2$ .
  - (d) [1+1 points.] Prove for  $n \geq 0$  that  $3|g_n$  if and only if  $n \equiv 1 \pmod 3$ .
  - (e) [1 point.] Today's date is 7112017. Show that  $g_{7112017}$  is divisible by 7.